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II. **Solution by HENRY HEATON, Atlantic, Iowa.**

The sum of the first two terms is $\frac{n+2}{2}$; of the first three, $\frac{(n+2)(n+4)}{2.4}$; of the first four, $\frac{(n+2)(n+4)(n+6)}{2.4.6}$; of the first r, $\frac{(n+2)(n+4)\ldots\ldots(n+2r-2)}{2.4\ldots\ldots(2r-2)}$. This may be rigorously demonstrated by induction. If $r=n$ the required sum is

$$\frac{(n+2)(n+4)\ \ldots\ldots\ldots\ (3n-2)}{2.4\ldots\ldots\ldots(2n-2)}.$$

Also solved by **L. E. Newcomb, J. Scheffer,** and **G. B. M. Zerr.**

248. **Proposed by S. A. COREY. Hiteman, Iowa.**

Prove that $\frac{6435}{2}\cdot\frac{161280^2}{929569}\left[1+\frac{1}{3^{16}}+\frac{1}{5^{16}}+\frac{1}{7^{16}}+\ldots\ldots\ldots\right]=\pi^{16}$.

I. **Solution by J. SCHEFFER, A. M., Hagerstown, Md.**

In books on higher algebra it is proved that

$$1+\frac{1}{3^{2n}}+\frac{1}{5^{2n}}+\frac{1}{7^{2n}}+\ldots\ldots=\frac{(2^{2n}-1)B_{2n-1}}{2n\,!}\cdot\frac{\pi^{2n}}{2},$$

B_{2n-1} being* the nth of Bernoulli's numbers.

$$\therefore 1+\frac{1}{3^{16}}+\frac{1}{5^{16}}+\frac{1}{7^{16}}+\ldots\ldots=\frac{(2^{16}-1)\,B_{15}}{16\,!}\cdot\frac{\pi^{16}}{2},$$

$$\therefore \pi^{16}=\frac{2\times16\,!}{(2^{16}-1)\,B_{15}}\left(1+\frac{1}{3^{16}}+\frac{1}{5^{16}}+\frac{1}{7^{16}}+\ldots\ldots\ldots\right).$$

Since $B_{15}=\frac{3617}{510}$, we find $\frac{2\times16\,!\times510}{(2^{16}-1)3617}$, which agrees with the coefficient given in the problem.

II. **Solution by G. B. M. ZERR. A. M., Ph. D., Parsons, W. Va.**

$$\left(1-\frac{4\theta^2}{\pi^2}\right)\left(1-\frac{4\theta^2}{3^2\pi^2}\right)\left(1-\frac{4\theta^2}{5^2\pi^2}\right)\ldots\ldots\ldots=\cos\theta=1-\frac{\theta^2}{2\,!}+\frac{\theta^4}{4\,!}-\ldots\ldots\ldots$$

$$\therefore \log\left(1-\frac{4\theta^2}{\pi^2}\right)+\log\left(1-\frac{4\theta^2}{3^2\pi^2}\right)+\log\left(1-\frac{4\theta^2}{5^2\pi^2}\right)+\ldots\ldots\ldots$$

$$=\log\left\{1-\left[\frac{\theta^2}{2\,!}-\frac{\theta^4}{4\,!}+\frac{\theta^6}{6\,!}-\frac{\theta^8}{8\,!}+\frac{\theta^{10}}{10\,!}-\frac{\theta^{12}}{12\,!}+\frac{\theta^{14}}{14\,!}-\frac{\theta^{16}}{16\,!}+\ldots\ldots\ldots\right]\right\}$$

$$=\log(1-A).$$

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*Chrystal's *Algebra*, Vol. II, Chapter 30, §15.

$$\therefore -\frac{4\theta^2}{\pi^2}\left(1+\frac{1}{3^2}+\frac{1}{5^2}+\ldots\right)-\frac{4^2\theta^4}{2}\left(1+\frac{1}{3^4}+\frac{1}{5^4}+\ldots\ldots\right)-\ldots\ldots$$

$$-\frac{4^8\theta^{16}}{8\pi^{16}}\left(1+\frac{1}{3^{16}}+\frac{1}{5^{16}}+\frac{1}{7^{16}}\right)=-A-\tfrac{1}{2}A^2-\ldots\ldots-\tfrac{1}{8}A^8-\ldots\ldots$$

Equating like coefficients of θ^{16} we get

$$\frac{8192}{\pi^{16}}\left(1+\frac{1}{3^{16}}+\frac{1}{5^{16}}+\frac{1}{7^{16}}+\ldots\ldots\right)=\frac{929569}{6435\times1260^2}.$$

$$\therefore \frac{6435}{2}\cdot\frac{161280^2}{929569}\left(1+\frac{1}{3^{16}}+\frac{1}{5^{16}}+\frac{1}{7^{16}}+\ldots\ldots\right)=\pi^{16}.$$

Also solved by F. Anderegg, and G. W. Greenwood.

249. Proposed by J. J. KEYES, Fogg High School, Nashville, Tenn.

Solve $x+y+z=5$, $x^2+y^2=z^2$, $x^3+y^3+z^3=8$.

Solution by M. R. BECK, Cleveland, Ohio.

$x+y+z=5$, or $x+y=5-z$.........(1), $x^2+y^2=z^2$.........(2),
$x^3+y^3+z^3=8$, or $(x+y)(x^2-xy+y^2)=8-z^3$.........(3).

From (1) and (2), we have $xy=\dfrac{25-10z}{2}$..........(4).

Substituting (1), (2), and (4) in (3), and solving, $z=\frac{47}{25}$. Substituting $z=\frac{47}{25}$ in (1) and (2) and solving, $x=\frac{39}{25}\mp\frac{7}{50}\sqrt{-34}$, and $y=\frac{39}{25}\pm\frac{7}{50}\sqrt{-34}$.

Also solved by Henry Heaton, A. H. Holmes, L. E. Newcomb, J. Scheffer, and G. B. M. Zerr.

AVERAGE AND PROBABILITY.

172. Proposed by J. EDWARD SANDERS, Reinersville, Ohio.

What is the average length of all straight lines that can be drawn within a given triangle?

II. Solution by HENRY HEATON, Atlantic, Iowa.

Let ABC be the triangle. Let x=length of one of the straight lines, and θ the angle it would make with the side AB if produced to meet it, θ being taken $>$ than the angle A and $<$ than $\pi-B$. At the distance $x\sin\theta$ from AB draw ED parallel to AB cutting AC in E and BC in D. Then the number of lines of length x making the angle θ with AB is equal to the number of points in the triangle DEC whose area is $\dfrac{\triangle\,(b\sin A-x\sin\theta)^2}{b^2\sin^2 A}$. Similarly, the number of lines of length x making the angle θ with AC is equal to the number of points in the triangle whose area is $\dfrac{\triangle\,(c\sin A-x\sin\theta)^2}{c^2\sin^2 A}$, and the number making same angle with the side BC is equal to the number of points in the triangle whose area is